[6] Bradshaw, P., "The Effect of Wind Tunnel Screens on 'Two-dimensional' Boundary Layers," Rept. 1085, 1963, National Physical Lab., Great Britain.

[7] Thwaites, B., "Approximate Calculation of the Laminar Boundary Layer," *Aeronautical Quarterly*, Vol. 1, 1949, pp. 245–80.

[8] Rott, N. and Crabtree, L. F., "Simplified Laminar Boundary Layer Calculations for Bodies of Revolution and for Yawed Wings," *Journal of the Aeronautical Sciences*, Vol. 19, No. 8, Aug. 1952, pp. 553–565.

Determination of Charge/Mass and Thrust from a Positively Charged Colloidal Beam

BRIAN MAKIN* AND ALFRED WILLIAM BRIGHT†
The University of Southampton, Southampton, England

1. Introduction

THE determination of the average charge/mass (q/m) of an aerosol or a charged colloidal beam at atmospheric pressure is an important parameter in any optimization study. A method is proposed which can be used for q/m measurements in either vacuum or at atmospheric pressure using standard laboratory equipment. The technique may prove to be useful in the performance analysis of some applications which use an electrically charged colloidal beam as the working medium.

A commercial process that has proved to be efficient and successful[1] is the process of electrastatic painting. A second field of interest is the electric propulsion of spacecraft. A charged colloidal beam with a relatively high charge/mass on each droplet (approximately 10^4 C/kg) is a serious contender for the present types of electric propulsion devices employing mercury, cesium, or potassium.[2] A singly charged cesium ion has a charge/mass of 7.26×10^5 coul/kg. A third more recent application in which charged aerosols could be important is in the generation of electricity (EGD).[3]

In all these applications the most important parameter which limits the efficiency is the ratio of charge/mass of each charged droplet. Many papers have been written on the dynamics of charged particles.[4,5] In the case of colloidal propulsion it is necessary to consider chemistry of the liquid during the formation of the droplet. This is a complex problem of charge neutralisation of the liquid ions at the wall of the nozzle or capillary needle. A survey of this problem has been given by Wineland and Hunter predicting criteria for stable operation.[6]

In this Note a method is put forward for determining the average charge/mass ratio for a charged aerosol which can be used at atmospheric pressure for any solvent, and in vacuo for liquids with a low vapour pressure where evaporation from a free surface is negligible. From a few parameters which can be measured accurately with standard equipment (chemical balance, oscilloscope, high-voltage generator), it is possible to determine the liquid thrust, the total thrust (liquid thrust plus liquid corona thrust), efficiency, and average exhaust velocity.

2. Experimental Arrangement and Theory

An aerosol generator was constructed from a glass tube of 5-mm o.d. with a glass nozzle of 0.45-mm-diam bore. Two similar tubes were fixed to a Teflon cup that was located on a low-friction pivot mounted vertically. The glass nozzles were positioned at right angles to the glass tubes and the assembly could rotate freely. The length of the radius arm was 12 cm. In these tests one nozzle was permanently sealed to eliminate any possible differences in efficiency of operation between the nozzles.

The Teflon cup was filled with liquid and an electrode was positioned below the liquid surface. When a high voltage was applied to the electrode, an aerosol was generated from the open nozzle. At atmospheric pressure the rotor revolved at speeds up to 12 rad/sec, with a wide range of liquids. Power was supplied from a 60 kv, 200 μa Brandenburg generator. The angular velocity and acceleration were measured by arranging the rotor to intersect a light beam. The equipment was located in a vacuum chamber and experiments were carried out at atmospheric pressure and subsequently at the ultimate 10^{-3} torr vacuum of the system. At atmospheric pressure the dominant process is expected to be the corona effect or electric wind. In vacuo the corona thrust would be minimized and hence any thrust developed would be produced from the charged colloidal beam.

In general, we can assume the thrust developed by the rotor consists of corona thrust (T_C), and from the acceleration of the charged colloid beam or liquid thrust (T_L).

In dynamic equilibrium

$$T_L + T_C = \mathrm{VI}\eta/\mathrm{r}\dot{\theta} \tag{1}$$

where VI is the input power, r the radius arm, $\dot{\theta}$ the equilibrium angular velocity, and η the efficiency.

The general equation of motion is

$$(T_L + T_C)\,\mathrm{r} - \mathrm{F} = J\ddot{\theta} \tag{2}$$

where F is the frictional couple of the pivot and J the moment of inertia of the rotor about a vertical axis through the electrode. At equilibrium $\ddot{\theta} = 0$ and F is determined at the equilibrium velocity $\dot{\theta}$ from a run down experiment.

A particle of charge q accelerating through a potential V volts will reach a terminal velocity v given by $V_q = \frac{1}{2}\,mv^2$. The thrust developed by the droplet (T_L) is equal to the rate of change of momentum; hence $T_L = \dot{m}v$, where $\dot{m}$ is the mass flow rate of the liquid. Hence the average charge/mass ratio $\langle q/m \rangle$ per droplet is

$$\langle \mathrm{q/m} \rangle = T_L^2/2\mathrm{V}\dot{m}^2 \tag{3}$$

Also, the power developed in accelerating the droplet is equal to the average work done ($\frac{1}{2}\,T_L v$). If we assume the efficiency

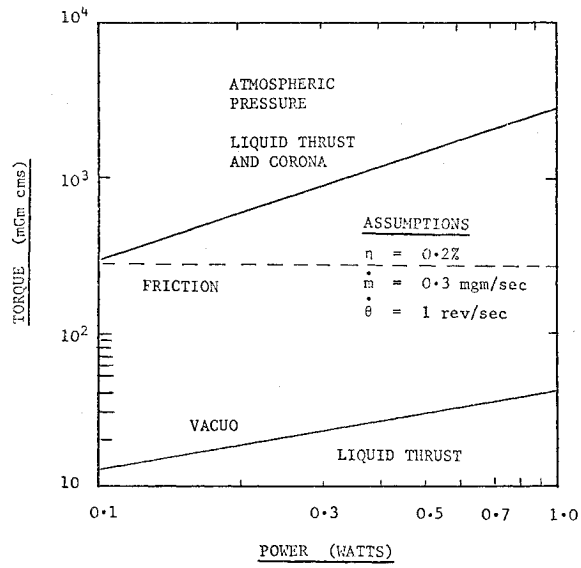

Fig. 1 Theoretical variation of torque with power.

Received December 16, 1968; revision received June 27, 1969.

* Lecturer, Department of Electrical Engineering.
† Reader, Department of Electrical Engineering.

Table 1 Experimental results: glycerol doped with $NiCl_2$, $ZnCl_2$, and NaI; liquid metal gallium/indium at atmospheric pressure

Liquid	Resistivity, kΩcm	Concentration, g/cm³	Power, w	$T_L + T_C$, mg	T_L, mg	η, %	v, m/sec	$q/m \times 10^{-3}$C/kg
Glycerol	2750	...	0.18	31.4	2.1	0.12	21.7	7.8
Glycerol doped with $NiCl_2$	2.47	0.1	0.08	29	1.3	0.29	36.3	33
	1.56	0.3	0.12	40	0.6	0.17	73.4	137
	0.87	0.4	0.1	26	0.9	0.21	46.6	53
Glycerol doped with $ZnCl_2$	11.2	0.1	0.15	22	1.4	0.15	33.2	25
	12.2	0.2	0.1	16	0.6	0.16	56.2	78
	17.8	0.3	0.1	24	0.5	0.26	105	279
	21.8	0.4	0.132	26	0.5	0.19	98.4	221
Glycerol doped with NaI	68.5	0.05	0.48	220	16.1	1.3	79.4	125
	30.4	0.1	0.65	270	19.6	1.3	87.4	152
	20.1	0.2	0.15	144	5.0	1.5	92.4	238
Gallium doped with indium		25% wt	0.39	120	1.57	0.9	460	3500

of producing thrust as given by (1) is equal to the efficiency of converting electrical power into a moving charged colloidal beam, then $\eta\, VI = \frac{1}{2}\, T_L\, v$;

$$\therefore \qquad \langle T_L \rangle = (2\eta \dot{m} VI)^{1/2} \qquad (4)$$

In this analysis the derived values are averages; no attempt has been made to include a distribution in aerosol particle size or the divergence of the aerosol due to space charge effects.

From (1) and (4), torque power curves have been computed and are shown in Fig. 1 for a rotor radius of 12 cm, together with a mass flow rate of 0.3 mg/sec, an efficiency of 0.2% and an equilibrium angular velocity of 1 revolution/sec. These assumptions are based on typical experimental results (see Table 1).

In the experimental tests, glycerol was used as the working fluid. Because of its low vapor pressure, it was ideally suited to operation in vacuo. Several additives were tested to investigate their effect on the charging mechanism. The data supplied here was obtained from different mixtures of glycerol/nickel chloride, glycerol/zinc chloride, glycerol/sodium iodide, and a liquid metal gallium/indium.

3. Experimental Results

Experimental determination of the frictional couple of the rotor bearing was found to vary slightly, the minimum value $\simeq$ 300 mgcm. In atmospheric tests the maximum power input levels were of the order 0.6 w. From Fig. 1 it can be seen that the liquid thrust by itself is not large enough to overcome the frictional resistance and this was verified by experiment. The efficiency was low for all the tests; the variation was from 0.1 to 1.5%. The results given in Table 1 were all obtained at atmospheric pressure with input powers of approximately 0.1 w and applied voltages of 20 kv (+ve).[7,8]

The experimental results show the effect of the additive concentration on the charging process. By increasing the concentration of $ZnCl_2$ from 0.1 g/cm³ to 0.4 g/cm³ the charge/mass passed through a maximum of 0.28 C/kg at a concentration of 0.3 g/cm³. The average droplet velocity $\langle v \rangle$ was derived from the rate of change of liquid momentum $\langle T_L \rangle / \dot{m}$. Undoped glycerol developed an exit velocity of 22 m/sec in comparison to the maximum velocity of 460 m/sec which occurred with the liquid metal gallium/indium. The liquid thrust component (T_L) was in all cases smaller than the corona thrust (T_C) and for the organic propellants the ratio T_C/T_L varied from 14 for glycerol to 66 for $NiCl_2$ at the concentration 0.3 g/cm³, and the total thrust was 31.4 and 40 mg, respectively.

Conclusions

From the experiments the average charge/mass ratio has been measured and the charging process was found to vary significantly with the choice and the concentration of the additive. From theory it was possible to infer the magnitudes of the liquid thrust component and these predicted the rotor would not rotate in vacuo, which was verified experimentally. The method proved, at atmospheric pressure, to be extremely simple to apply and indicated the importance of the doping additive that was similar to that obtained by Wineland and Hunter whose measurements were made in vacuo using a quadrupole mass spectrometer.

The effect of adding $ZnCl_2$ to glycerol was found to improve the charging efficiency varying from $\langle q/m \rangle$ of 0.008 coul/kg to 0.28 coul/kg. This was accompanied by a decrease in the resistivity of the undoped glycerol to the optimum charging value of 1.78×10^4 Ωcm. Wineland and Hunter similarly found there was an optimum concentration of $ZnCl_2$ in glycerol, which when used in vacuo gave a $\langle q/m \rangle$ of 3000 coul/kg at a resistivity of 6×10^4 Ωcm. The thrusts obtained from glycerol doped with NaI were considerably larger than any of the other propellants. A maximum liquid thrust of 19 mg was produced with a concentration of 0.1 g/cm³. This result is in general agreement with other workers who find NaI to be one of the best dopants when the concentration is approximately 0.2 g/cm³.

With the liquid metal a q/m of 3.5 coul/kg was obtained at the highest exhaust velocity of 460 m/sec. This propellant has been tested in vacuo[9] and specific charges of 7×10^5 coul/kg were recorded suggesting the formation of ions.

Using the Rayleigh criterion for droplet stability,[4] the average droplet size at the optimum charging conditions with glycerol $ZnCl_2$ was 5μ. These results are in close agreement with what has been found in electrostatic paint spraying. The best practical results are generally particle sizes $\simeq 1\mu$ and $q/m \simeq 0.1$ coul/kg, and a resistivity between 2×10^6 and $2 \simeq 10^7$ Ωcm. These conditions are approximately those obtained for undoped glycerol. From these experiments we have shown that a significant improvement can be obtained in the charging processes at atmospheric pressure, if the liquid conductivity is increased.

From the stability criteria for charge neutralization,[6] an improvement in the charging efficiency of the aerosol generator would be expected if the polarity was reversed to negative. It has been shown that the work functions $\phi+$, $\phi-$ for either +ve or −ve polarity must be related to the level of the ion in solution.

In the case of $ZnCl_2$, a positive polarity required complete neutralisation of the cations (Zn^+) for maximum charging and $\phi- < 6.29$ ev. With a negative polarity the anions (Cl^-) are neutralised and $\phi+ > 5.23$ ev. In both inequalities a double layer potential of 1 ev has been assumed. Hence, it can be concluded that a glass capillary would be ideally suited for operation at negative potentials with glycerol/$ZnCl_2$ solutions and this may prove sufficient to achieve larger liquid thrusts and the possibility of measuring q/m in vacuo.

References

[1] Oesterle, K. M., "Bildung von organischen Schutzuberzugen durch elektrostatischen Auftrag von Lacken und Pulvern," *Proceedings of the International Conference on Protection against Corrosion by Metal Finishing*, Forster Verlag, Zurich, 1967, pp. 265–282.

[2] Schultz, R. D. and Wilch, R. E., "Electrical Propulsion with Colloidal Materials," *Advanced Propulsion Techniques*, edited by S. S. Penner, Pergamon, New York, 1961, pp. 98–115.

[3] Hasinger, S. et al., "Electro-Fluid Power Generation," *Combustion and Propulsion VI AGARD Colloquium*, edited by H. M. De Graff et al., Gordon and Breach, New York,1967, pp. 539–630.

[4] Rayleigh, I.W.S., "On the Equilibrium of Liquid Conducting Masses Charged with Electricity," *Philosophical Magazine*, Vol. 14, 1882, pp. 184–186.

[5] Hogan, J. J., "Parameters Influencing Charge-to-Mass Ratio of Electrically Sprayed Liquid Particles," C.P.R.L., -2-63, Electrical Engineering Dept., Univ. of Illinois, Urbana, Ill.

[6] Wineland, S. H. and Hunter, R. E., "Negatively Charged Colloid Generation Research," AIAA Paper 66–251, San Diego, Calif., 1966.

[7] Lee, D. J., "Electrostatic Thrusters," Rept., July 1968, Dept. of Electrical Engineering, Univ. of Southampton, Southampton, England.

[8] Green, P. E. S., "Charged Particle Studies for Colloid Propulsion," Rept., July 1969, Dept. of Electrical Engineering, Univ. of Southampton, Southampton, England.

[9] Swatick, D. S. and Hendricks, C. D., "Production of Ions by Electrohydrodynamic Spraying Techniques," *AIAA Journal*, Vol. 6, No. 8, Aug. 1968, pp. 1596–1597.

Dispersion Analysis by the Simultaneous Application of Tolerances

Abram Lazarus*

The Aerospace Corporation, El Segundo, Calif.

Introduction

One well-known technique for accomplishing a dispersion analysis is the so-called root-sum-square (RSS) method. Normally, one conducts the RSS method by first calculating a nominal case and then successively varying one parameter at a time, while observing the deviation from nominal caused by the perturbation of that particular parameter. These deviations from nominal are then root-sum-squared to obtain the expected resultant deviation under toleranced conditions. The drawback of this procedure is that it requires n separate analyses for n given parameters.

In this paper it is shown that, under most conditions, it is feasible to perform a single analysis in which the various parameters are toleranced simultaneously, and accomplish essentially the same end result that the RSS method yields.

Conventional Root-Sum-Square (RSS) Approach

Assume that the function Y, which is being investigated for the effect of tolerances, is a function of the parameters $X_1, X_2, \ldots, X_n$. The approximate dispersion in Y due to parameter variations can be expressed by the linear expansion

$$\Delta Y = \sum_{i=1}^{n} K_i \Delta X_i \tag{1}$$

where the K_i terms are usually referred to as "influence coefficients" and the ΔX_i terms represent the parameter deviations from nominal.

Since the X_i parameters are subject to tolerances, they can be considered as random variables that are statistically distributed about their nominal values. If the X_i parameters are independent, then the k-sigma dispersion in ΔY is given by the familiar RSS relationship

$$k\sigma_Y = \left[\sum_{i=1}^{n} (K_i k\sigma_{X_i})^2\right]^{1/2} \tag{2}$$

where the σ_{X_i} values represent the standard deviations of the individual ΔX_i distributions.

To apply Eq. (2), we must first evaluate the K_i influence coefficients. Normally, the relationship between Y and its parameters is known only implicitly; therefore, one usually evaluates the influence coefficients by varying the parameters one at a time and noting the corresponding dispersions in Y. Thus, this procedure requires n separate analyses for n given parameters.

Proposed Approach

Assumptions

In addition to the usual assumptions of linearity and statistical independence required for the RSS method, the proposed approach is predicated on the following assumptions: 1) the directional characteristics of the tolerance effects, i.e., the signs associated with the products of the influence coefficients and their corresponding tolerances, are known; and 2) the significant tolerance effects outnumber the insignificant tolerance effects.

The first assumption is vital to the proposed dispersion analysis technique, since it depends on applying the tolerances such that their effects are all in the same direction. Satisfaction of this assumption may require some previous experience or some a priori reasoning based on the physics of the problem.

The second assumption is aimed at achieving good accuracy, since the accuracy of the method depends on constraining the value of a statistical parameter known as the coefficient of variation† between zero and unity. Should a preponderance of small terms be toleranced in proportion to the significant terms, then it is possible for the coefficient of variation to exceed the latter constraint. From a practical standpoint, however, this assumption will tend to be satisfied automatically, as the analyst will usually exclude negligible effects from the analysis.

Description of method

The approach suggested in this paper is to perform a single analysis and obtain essentially the same result for $k\sigma_Y$ as would normally be obtained from the conventional RSS approach. The procedure is simply to apply appropriately scaled tolerances simultaneously, in the proper direction, and note the resultant dispersion in Y due to the combined effect of the tolerances.

Referring to Eq. (1), note that if all the tolerance effects were in at the same time and in the same direction, say, positive, then all the parameter deviations would contribute uniquely to an increase in ΔY. With this in mind, we can express the resulting dispersion in Y by a modified form of Eq. (1) as

$$\Delta Y = \sum_{i=1}^{n} |K_i \Delta X_i| \tag{3}$$

where the absolute values denote that all the terms are forced positive by proper selections of signs associated with the tolerances.

Received February 27, 1969; revision received May 19, 1969. This work was supported by Air Force Contract F04701-68-C-0200. The author wishes to acknowledge the helpful comments of T. C. Coffey.

* Member of the Technical Staff, Guidance Analysis Department. Member AIAA.

† Defined as the ratio of the standard deviation to the mean of a statistical sample.[1]